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Calculation of Nuclear Matrix Elements for the Two-Neutrino Double Beta Decay of ^{128,130}Te Isotopes

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Abstract

In this study, the nuclear matrix elements for the two-neutrino double beta decay ($2\nu\beta\beta$) of ^{128,130} Te isotopes have been calculated by using Tamm-Dancoff Approximation (TDA) with the theory of residues by considering the charge-exchange spin-spin interactions in the particle-hole channel. Calculations have been performed for the spherical case of the nuclei on the basis of single particle energies calculated in the Hartre Fock Approximation with Sykrme-III forces. The evaluated results show that this type of single particle base can be used as an alternative approximation for calculation of nuclear matrix elements of double beta decay.

Key Words: *Hartree-Fock Approximation; Sykrme Forces; Random Phase Approximation; Tamm-Dancoff Approximation; Double Beta Decay*

Özet

Bu çalışmada, ^{128,130} Te izotoplarının iki-nötrinolu çift bozunumu ($2\nu\beta\beta$) için nükleer matris elemanları, yük alışverişli etkileşmeyi parçacık-deşik kanalında dikkate alarak, Tamm-Dancoff Yaklaşımı'nın (TDA) rezidüler teoremiyle birlikte kullanılmasıyla hesaplandı. Hesaplamalar, küresel çekirdek durumunda, Hartre Fock Yaklaşımı'nı Sykrme-III kuvvetleri ile birlikte kullanarak elde edilen tek parçacık enerjileri bazında yapıldı.. Elde edilen sonuçlar bu tip tek parçacık bazının çift beta bozunumu nükleer matris elemanı hesabında alternatif bir yaklaşım olarak kullanılabileceğini göstermektedir.

Anahtar Kelimeler: *Hartree-Fock Yaklaşımı; Sykrme Kuvvetleri; Keyfi Faz Yaklaşımı; Tamm-Dancoff Yaklaşımı; Çift Beta Bozunumu*

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1. Introduction

Nuclear double-beta decay *($\beta\beta$)* is one of the rarest processes in nature which is observed between even-even nuclei. It can be observed only when the single β-decay is strongly suppressed due to a large change of spin or forbidden energetically. Although the main interest in *($\beta\beta$)*-decay is related to the neutrinoless mode *($0\nu\beta\beta$)* in order to bring different aspects to elementary particle physics beyond the standard model, considerable efforts are made to investigate the ordinary allowed second order weak decay two neutrino mode *($2\nu\beta\beta$)* (Moe et al 1994, Suhonen et al 1998, Faessler et al 1998). Accumulation of experimental information for processes (transitions to the ground and excited states), promotes a better understanding of nuclear part of $\beta\beta$-decay and allows one to check theoretical schemes of nuclear matrix element calculations for the two-neutrino mode. The reliable evaluation of the $2\nu\beta\beta$-decay nuclear matrix elements is necessary to gain confidence for calculation of $0\nu\beta\beta$–decay nuclear matrix element.

Over the past few years, the nuclear matrix element has been calculated mainly in three types of models, namely Nuclear Shell Model (NSM) and its variants, the Quasiparticle Random Phase Approximation (QRPA) and its extensions and the alternative models. The details of these models have been discussed by Suhonen et al., (Suhonen et al., 1998) and Faessler et al., (Faessler et al 1998). NSM attempts to solve the nuclear many body problem as exactly as possible. Hence, it is the best choice for the calculation of the nuclear matrix elements. Despite of the advantages of calculation techniques only a limited set of single particle states can be used in NSM. QRPA is the nuclear much body method most widely used to deal with the nuclear structure aspects of the double beta decay process. The QRPA has been found successful in explaining the quenching of the $0\nu\beta\beta$-decay nuclear matrix element and bring them into closer agreement with experimental values. But despite this success the QRPA approach to double beta decay has some shortcomings. The main problem is that results are extremely sensitivity to the renormalization of the attractive particle-particle component of residual interaction, which is large part responsible for suppressing calculated two-neutrino decay rate. Including proton-neutron pairing, several alternations have been made in QRPA. However, none of these alternations have changed the rapid variation of calculated $2\nu\beta\beta$-decay matrix element with the increasing strength of the particle-particle interaction. Recently there have been made some corrections on quasi-boson approximation (Suhonen et al 1998) leading to a violation of the Pauli Exclusion

Principle. Alternative models, as the operator expansion method (OEM), the broken SU (4) symmetry, two vacua RPA, the pseudo SU (3) and single state dominance hypothesis (SSDH) have their own problems (Suhonen et al., 1998). Recently, deformation structure in nuclei has been taken into account in calculation of nuclear matrix element (Selam, et al., 2003 and Chandra et al., 2005). In present, the calculated matrix elements are far away from the experimental results.

In calculations of nuclear matrix elements, the wave functions and the energy eigenvalues of 1^+ state in intermediate odd-odd nucleus should be considered. Since the 1^+ state has a high density, in order to find the eigenvalues, it is necessary to solve complex equations. The problem can be solved analytically by using the theory of residues and counter integrals in Tamm-Dancoff Approximation (TDA) where the quasiparticle correlations are neglected in the ground state. Balaev et al. calculated nuclear matrix elements for selected nuclei in TDA using the theory of residues and counter integrals (Balaev et al., 1990). One should consider that the TDA is a rudimentary approximation because of neglecting quasiparticle correlations in ground states, but it brings simplicity to calculate nuclear matrix element, and reliability of any single particle energy basis can be tested.

Bobyk and Kaminski showed that nuclear matrix element is strongly dependent on the choice of single particle energy basis (Bobyk et al., 1985). The single particle energies evaluated in the Hartree-Fock Approximation with Sykrme III Forces (HF+SIII) have not been used so far in calculation of double beta decay matrix element. Thus, in this work, it is aimed at testing the reliability of this single particle energy base for calculation of $M_{2\nu}$ and calculations have been done for ^{128,130} Te isotopes by using the TDA with the theory of residues by considering the charge-exchange spin-spin interaction among nucleons in the particle-hole channel. In the next section, we give the theoretical description used in our analysis and then in section-3, we present our results and compare them with the experimental findings. Finally, section-4 is devoted to our conclusion.

2. Theory

The half-life of $2\nu\beta\beta$ decay mode for $0^+ \rightarrow 0^+$ transition can be written in the form of (Haxton et al., 1984, Doi et al., 1985, Vergados et al., 1976)

$$[T_{1/2}]^{-1} = F^{2\nu} \left| M_{GT}^{2\nu} \right|^2, \tag{1}$$

where $F^{2\nu}$ is the exactly calculable phase space integral containing the entire relevant constant, and $M_{GT}^{2\nu}$ is the nuclear matrix element (there is no Fermi part, due to isospin conservation). This part is given by (Balaev et al., 1990)

$$M_{GT}^{2\nu} = \sum_{n} \frac{\langle (A,Z+2)|\vec{\sigma}\tau_{-}|n\rangle\langle n|\vec{\sigma}\tau_{-}|(A,Z)\rangle}{\omega_n + W/2}, \qquad (2)$$

where $|A,Z+2\rangle(|A,Z\rangle)$ are the 0^+ ground states of the final (initial) even-even nuclei, $|n\rangle$ are the 1^+ state in the intermediate odd-odd nucleus of energy ω_n and W is the 2β-decay energy, $\vec{\sigma}$ and $\vec{\tau}$ are and are the Pauli spin operator and isospin operator successively? Table 1 show the W and $F^{2\nu}$ values for $^{128}Te \rightarrow ^{128}Xe$ and $^{130}Te \rightarrow ^{130}Xe$ transitions (Aliev, T.M., et al., 1989).

Table 1 W and $F^{2\nu}$ values for $^{128}Te \rightarrow ^{128}Xe$ and $^{130}Te \rightarrow ^{130}Xe$ Transitions

Transitions	W (MeV)	$F^{2\nu}$ (year^{-1}MeV^{-1})
$^{128}Te \rightarrow ^{128}Xe$	1.891	2.16×10^{-22}
$^{130}Te \rightarrow ^{130}Xe$	3.555	1.23×10^{-18}

2.1. The Nuclear Model

Consider a system of nucleus in spherical symmetric mean field interacting via pairing and effective charge-exchange spin-isospin interaction among the nucleons. Hamiltonian of the system can be written as

$$H = H_{av} + H_{pair} + V_{\sigma\tau}, \qquad (3)$$

where

$$H_{av} = \sum_{jm} E_j a_{jm}^{+} a_{jm}, \qquad (4)$$

$$H_{pair} = -G \sum_{jm>} a_{jm}^{+} a_{j-m}^{+} a_{j'-m'} a_{j'm'}, \qquad (5)$$

$$H_0 = H_{av} + H_{pair} \qquad (6)$$

$$V_{\sigma\tau} = \frac{1}{2}\chi_{\beta} \sum_{i \neq j} \vec{\sigma}_i \vec{\sigma}_j \vec{\tau}_i \vec{\tau}_j. \qquad (7)$$

Here, Eq. (4) represents single particle Hamiltonian with the energy of E_j, a^{+}_{jm} (a_{jm}) are single particle creation and annihilation operators. The interaction, which causes pairing superfluid correlations, is represented by H_{pair} given by Eq. (5). Here, G is the pairing constant and takes different values for protons and neutrons. H0 describes the average motion of nucleons. Since the quasiparticle operator coincides with the particle (hole) operator for the states far above (below) the Fermi Level, one can express H_0 using quasiparticle creation and annihilation operators as (Soloviev, V.G., 1976)

$$H_0 = H_{sqp} = \sum_{jm} \varepsilon_{j_n} \alpha^{+}_{j_n m_n} + \alpha_{j_n m_n} + \sum_{jm} \varepsilon_{j_p} \alpha^{+}_{j_p m_p} \alpha_{j_p m_p} \,. \tag{8}$$

Here α^+ (α) are the quasiparticle creation (annihilation) operators with spin and parity 1^+; these operators are defined as

$$a^{+}_{jm} = u_j \alpha^{+}_{j-m} + (-1)^{j-m} v_j \alpha_{jm} \,, \tag{9 a}$$

$$a_{jm} = u_j \alpha_{j-m} + (-1)^{j-m} v_j a^{+}_{jm} \,. \tag{9 b}$$

Here, u_{j} and v_j coefficients are given by

$$v_j^2 = \frac{1}{2}\left[1 - \frac{E_j - \lambda}{\varepsilon_i}\right]; u_i^2 = 1 - v_j^2 \,, \tag{10}$$

where ε_j is the quasiparticle energy which is the eigenvalue of Hamiltonian H_{sqp} and given by

$$\varepsilon_j = \sqrt{\Delta^2 + (E_j - \lambda\,)^2} \,, \tag{11}$$

where Δ denotes the energy gap parameter and is determined phenomenological from odd-even mass differences through a symmetric five terms formula involving the experimental binding energies (Möller, P. and Nix, J.R, 1992)

$$\Delta_n = \frac{1}{8}[B(N-2,Z) - 4B(N-1,Z) + 6B(N,Z) - 4B(N+1,Z) + B(N+2,Z)]. \tag{12}$$

A similar expression is found for proton gap Δ_p by changing N by Z and vice versa. Ej is the single particle energy of nucleons and λ is Fermi energy and can be calculated using the number equation, which is given by (Soloviev, V.G., 1976)

$$N=\sum_j(1+\frac{1}{2})\left\{1-\frac{E_j-\lambda}{(E_j-\lambda)^2+\Delta^2}\right\}. \quad (13)$$

In Eq. (3), $V_{\sigma\tau}$ is the charge-exchange Gamow-Teller interaction which is responsible for the β-transitions in odd-odd nucleus in the particle-hole channel. Here, $\vec{\sigma}$ and $\vec{\tau}$ Pauli spin operator and isospin operator. The part of $V_{\sigma\tau}$ which generates the collective 1^+ state in odd-odd nuclei given as (Aytekin, H., and Kuliev, A.A., 1996)

$$V_{ph}^{coll}=2\chi_\beta\sum_\mu T_\mu^{B+}T_\mu^B \quad (14)$$

where

$$T_\mu^B=\sum_{np}(\bar{b}_{np}C_{np}(\mu)+(-1)^{\mu+1}b_{np}C_{np}^+(-\mu);\ T_\mu^{B+}=(T_\mu^B)^+ \quad (15)$$

$$C_{np}(\mu)\equiv\sum_{m_n,m_p}\sqrt{\frac{3}{2J_n+1}}(-1)^{J_p-m_p}\left\langle J_pm_p1\mu|J_nm_n\right\rangle\alpha_{j_p-m_p}\alpha_{j_nm_n}, \quad (16)$$

$$b_{np}=\frac{1}{\sqrt{3}}u_{j_p}v_{j_n}\left\langle J_n\|\vec{\sigma}\|J_p\right\rangle\ ;\bar{b}_{np}=\frac{1}{\sqrt{3}}u_{j_n}v_{j_p}\left\langle J_n\|\vec{\sigma}\|J_p\right\rangle. \quad (17)$$

Here, $C_{np}^+(C_{np})$ are the bosonic operators representing the creation (annihilation) of neutron-proton pairs. Here, $\left\langle J_n\|\vec{\sigma}\|J_p\right\rangle$ is reduced matrix element and given by (Suhonen et al., 1988)

$$\left\langle J_n\|\vec{\sigma}\|J_p\right\rangle=-\sqrt{6}\delta_{\alpha_p\alpha_n}\delta_{l_pl_n}(-1)^{j_p+l_n+1/2}W(\frac{1}{2}\frac{1}{2}j_nj_p;1l_n) \quad (18)$$

Where α_p and α_n denote all the other quantum numbers except the angular momentum quantum numbers.

2.2. QRPA and Collective 1+ States in Odd-Odd Nuclei

In the QRPA, collective 1^+ state is built of a linear combination of two-quasiparticle states. The wave function of this state is looked upon as one-phonon function and is given in the form (Aytekin, H. and Kuliev, A.A., 1996)

$$\left|\psi_n\right\rangle=Q_n^+\left|\psi_0\right\rangle=\sum_{np}(\psi_{np}^nC_{np}^+(\mu)+(-1)^\mu C_{np}\varphi_{np}^n(-\mu))\left|\psi_0\right\rangle, \quad (19)$$

where Q_n^+ is the phonon creation operator, $|\psi_0\rangle$ is the phonon vacuum which corresponds to the ground states of parent even-even nucleus. The two quasiparticle amplitudes ψ_{np}^n and φ_{np}^n are normalized by

$$\sum_{np}\left[(\psi_{np}^n)^2 - (\varphi_{np}^n)^2\right] = 1 . \tag{20}$$

Following the conventional procedure of RPA and solving the equation motion

$$\left[H_{sqp} + V_{ph}^{coll}, Q_n^+\right]\psi_0\rangle = \psi_n Q_n^+ |\psi_0\rangle , \tag{21}$$

we obtain the dispersion relation for the excitation energy ω_n of 1^+ state in odd-odd nuclei as

$$D_1(\omega_n)D_2(\omega_n) - (2\chi_{ph}D(\omega_n))^2 = 0 , \tag{22}$$

where $D(\omega)$, $D_1(\omega_n), D_2(\omega_n)$ and $D_3(\omega_n)$ functions are defined as

$$D_1(\omega_n) = 1 + 2\chi_{ph}\sum_{np}\left(\frac{b_{np}^2}{\varepsilon_{np} - \omega_n} + \frac{\bar{b}_{np}^2}{\varepsilon_{np} + \omega_n}\right) , \tag{23}$$

$$D_2(\omega_n = 1 + 2\chi_{ph}\sum_{np}\left(\frac{\bar{b}_{np}^2}{\varepsilon_{np} - \omega_n} + \frac{b_{np}^2}{\varepsilon_{np} + \omega_n}\right) , \tag{24}$$

$$D_3(\omega_n) = \sum_{np} b_{np}\bar{b}_{np}\left(\frac{1}{\varepsilon_{np} - \omega_n} + \frac{1}{\varepsilon_{np} + \omega_n}\right) , \tag{25}$$

$$D(\omega) = D_1(\omega_n)D_2(\omega_n) - (2\chi_{ph}D_3(\omega_n))^2 \tag{26}$$

where $\varepsilon_{np} = \varepsilon_n + \varepsilon_p$ is the two quasiparticle energy for neutron-proton pairs.

Using the normalization form (20), we can easily obtain expressions for two quasiparticles amplitudes ψ_{np}^n and φ_{np}^n :

$$\psi_{np}^n = -\frac{1}{\sqrt{Y_n^{ph}}}\left(\frac{\bar{b}_{np} + Y_n^{ph} b_{np}}{\varepsilon_{np} - \omega_n}\right),\ \varphi_{np}^n = \frac{1}{\sqrt{Y_n^{ph}}}\left(\frac{b_{np} + L_n^{ph}\bar{b}_{np}}{\varepsilon_{np} + \omega_n}\right. , \tag{27}$$

where

$$Y_n = \sum_{np}\left\{\frac{(\bar{b}_{np} + L_n b_{np})^2}{(\varepsilon_{np} - \omega_n)^2} - \frac{(b_{np} + L_n \bar{b}_{np})^2}{(\varepsilon_{np} + \omega_n)^2}\right\}, \tag{28}$$

$$L_n = \frac{2\chi_{ph} D_3(\omega_n)}{1 + 2\chi_{ph} D_1(\omega_n)} = -\frac{1 + 2\chi_{pp} D_2(\omega_n)}{2\chi_{pp} D_3(\omega_n)}. \tag{29}$$

If we discard factors containing $1/(\varepsilon_{np} + \omega_i)$ in (23)-(28), we obtain the formulas corresponding to the Tamm-Dancoff Approximation (TDA).

2.3. Nuclear Matrix Elements

In this study, the ground states of the parent and daughter nuclei are assumed to be same, i.e. $(|\psi_i\rangle = |\psi_f\rangle = |\psi_0\rangle)$. Matrix elements of beta transitions given in Eq. (2) take the form of

$$M_{\beta^-}^{n\downarrow} = \langle \psi_f(A, Z+2) | \vec{\sigma}\tau_- | 1_n^+ \rangle = \frac{1}{2\chi_{ph}\sqrt{Y(\omega_n)}}, \tag{30}$$

$$M_{\beta^-}^{n\uparrow} = \langle 1_n^+ | \vec{\sigma}\tau_+ | \psi_i(A, Z) \rangle = \frac{L(\omega_n)}{2\chi_{ph}\sqrt{Y(\omega_n)}}. \tag{31}$$

If we use (28) and (29), we find that the general expression for $M_{GT}^{2\nu}$, given in (2), in the form of

$$M_{GT}^{2\nu} = 3\sum_n \frac{M_{\beta^-}^{n\downarrow} M_{\beta^-}^{n\uparrow}}{\omega_n + W/2}. \tag{32}$$

Now the basic theorem of residues allows us to write the expressions for (32) (Balaev et al., 1990) in the following form:

$$M_{GT}^{2\nu} = 3\frac{D_3(-W/2)}{D(-W/2)}, \tag{33}$$

Which is depended on the decay energy of W and the interaction constant χ_{ph}.

3. Numerical Results and Discussions

The single-particle energies used in the numerical calculations were computed using the Hartree-Fock Approximation with Skyrme-III forces described in the reference (Negele et al., 1972). In this approximation, the wave functions of proton and neutron for a given level have been calculated as equal to each other.We considered the ^{40}Ca nucleus as core and took the same single-particle basis both for protons and neutrons shown in Table 1. We have first calculated Δ_n and Δ_p using formula (12) with the aid of experimental binding energies (Audi et al., 1993). Using these parameters λ_n and λ_p are calculated by Eq. (13). Following the equations (11), (10), (18), (17, (25), (26) and (33) in order, $M_{GT}^{2\nu}$ is calculated.

Table 2. Single-particle energies in units of MeV

Shell	^{128}Te		^{130}Te	
	p	n	p	n
$1f_{7/2}$	-21.298	-26.430	-21.9558	-26.394
$1f_{5/2}$	-18.090	-23.024	-18.717	-23.259
$2p_{3/2}$	-15.436	-20.725	-16.080	-20.710
$2p_{1/2}$	-14.144	-19.384	-14.798	-19.362
$1g_{9/2}$	-12.352	-17.470	-13.080	-17.459
$1g_{7/2}$	-7.141	-12.104	-7.878	-12.259
$2d_{5/2}$	-5.412	-11.015	-6.066	-11.069
$2d_{3/2}$	- 3.394	-8.738	-4.061	-8.797
$3s_{1/2}$	-2.696	-8.677	-3.315	-8.694
$1h_{11/2}$	-3.013	-8.189	-3.720	-8.219

In this study, $2\nu2\beta$ decay nuclear matrix elements of ^{128}Te and ^{130}Te isotopes have been calculated in TDA using the theory of residues and counter integrals. Since, Balaev et al. (Balaev et al., 1990) used the same method for their calculation we present the results of our analyses in table 2 in comparison with the results of Balaev et al., and experimental results. The experimental data for $^{128}Te \rightarrow ^{128}Xe$ transition was measured by Bernatowichz et al., (Bernatowichz et al., 1992) and for $^{130}Te \rightarrow ^{130}Xe$ transition the data was measured by Bernatowichz et al., (Bernatowichz et al., 1992) and Takaoka et al (Takaoka et al., 1996). For $^{128,130}Te \rightarrow ^{128,130}Xe$ transitions, our results obtained by using the TDA are shown in the third row of table 3. The second row shows the results of Balaev et al., (Balaev et al., 1990) obtained also for by using the wood-Saxon potential in TDA. The last row in the same table shows the experimental results. Our results approach to the experimental ones better than the results of Balaev et al.

We also show the dependence of $M_{GT}^{2\nu}$ on χ_{ph} for ^{128,130}Te isotopes in Figure 1. We have taken $\chi_{ph} = 0.15 MeV$ n our analyses. When $\chi_{ph} = 0$, the numerical values of $M_{GT}^{2\nu}$ correspond to the quasiparticle model. As χ_{ph} increases, the quantity $M_{GT}^{2\nu}$ declines and for large quantities tends quadraticaly to the zero.

Table 2. Nuclear Matrix Element $M^{2\nu}_{GT}$ (MeV^{-1}) for $2\nu 2\beta$

Transitions	$^{128}Te \to {}^{128}Xe$	$^{130}Te \to {}^{130}Xe$
(Balaev et al., 1990)	0.90	0.88
This work	0.372	0.283
Experiment	0.025 Ref. (Bernatowichz et al., 1992)	0.017;0.037 Ref. (Takaoka et al., 1996)

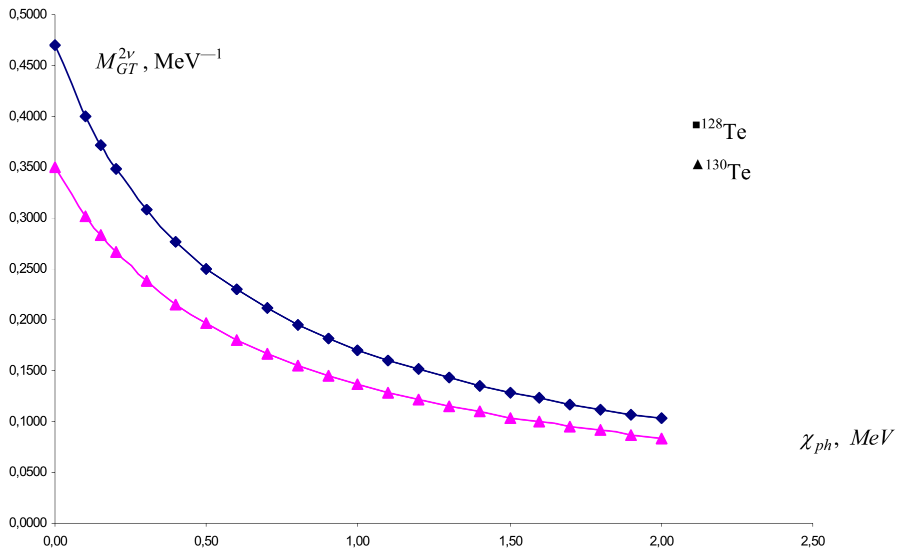

Figure 1. *Dependence of nuclear matrix elements* $M^{2\nu}_{GT}$ on χ_{ph} *parameter*

4. Conclusion

We have calculated $M^{2\nu}_{GT}$ using TDA with the theory of residues at the single particle energy base, which was evaluated in the Hartree-Fock approximation with Skyrme-III forces. Our findings are better than that of Balaev et al. (Balaev et al 1990), which were evaluated in the same method, but using the base of Wood-Saxon Potential. In the light of our results, we may conclude that one can use the single particle energies, which is evaluated in the Hartree-Fock Approximation with Skyrme-III forces, in realiability in calculation for nuclear matrix elements of two-neutrino double beta decay in the developed models described in section 1.

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